3D theories and 3-mfds

2024 弦论、引力与引力波研讨会

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w/ P. Sułkowski 2302.13371, 2310.07624, and work in progress

Motivation

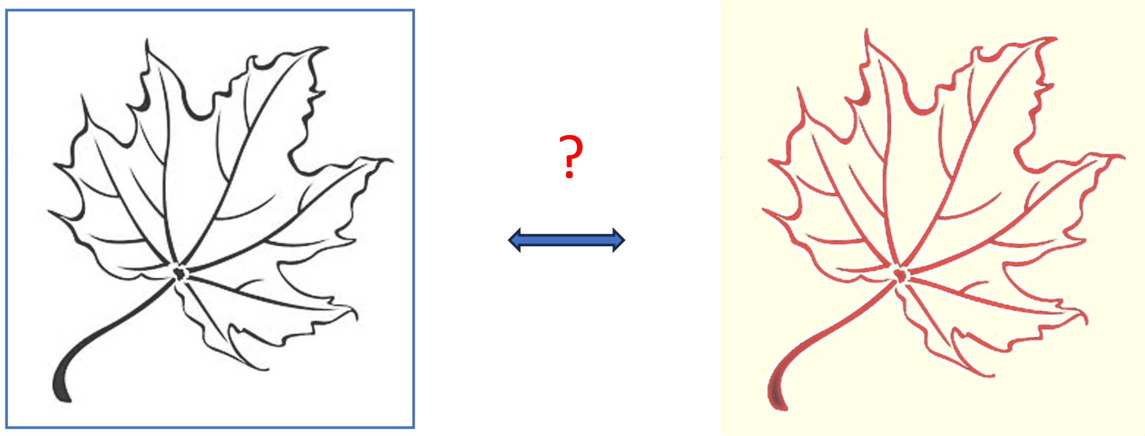
- 3d theories with 4 supercharges are not understood yet.
- String theory could construct gauge theories, such as M5-branes on 3-manifolds. However, we are still very far away from the final answer.

Tools

- 3d N=2 gauge theories: dualities, gauging
- String theories: M-theory/IIB duality, 3d brane webs
- 3-manifolds: surgery, Kirby moves

- Instead of non-abelian theories, we find the abelian theories are already powerful engough to detect 3-manifolds.

- In this talk, we will show a remarkable match between structures:



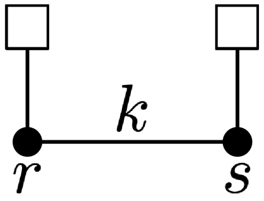
3d theories ⟷ 3-manifolds

- Hence, 3-manifolds can be used to understand 3d gauge theories

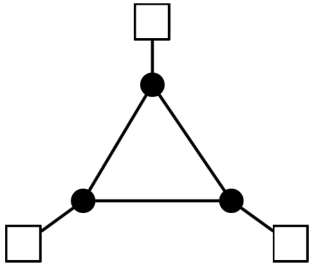
3d abelian theories -- plumbing theories

- A new quiver diagram (plumbing graphs): $\bullet_k$ $\quad U(1)_k \quad$ $\square$ $\quad 1\ \Phi$

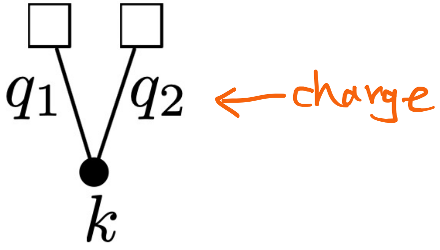
Mixed CS levels:

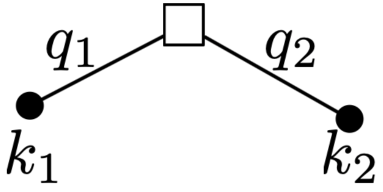


$C_{ij} = \begin{bmatrix} r & k \\ k & s \end{bmatrix}$



Charges:

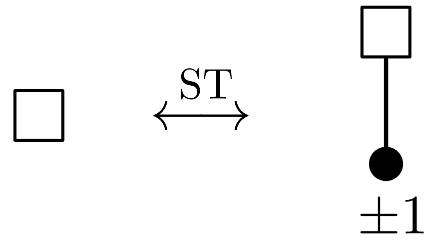




3d dualities

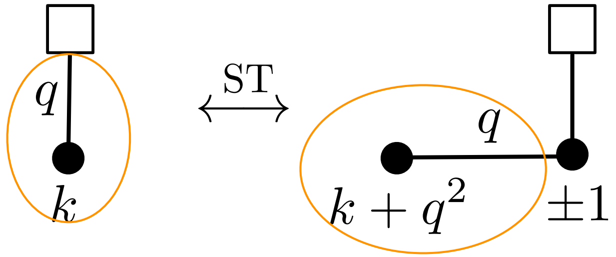
- Gauge the mirror duality -> ST-moves

$$1\Phi \leftrightarrow U(1) + 1\Phi$$



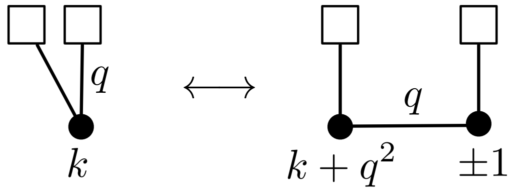
Flavor symmetry: $U(1)_F \longleftrightarrow U(1)_T$

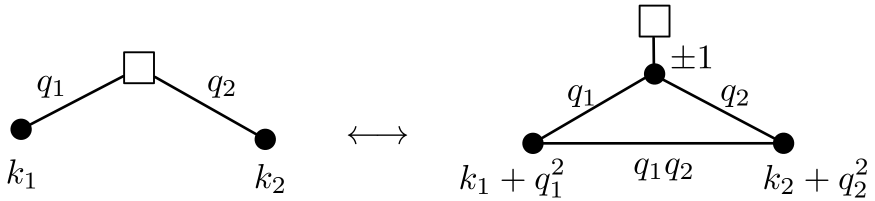
Gauge the U(1) :

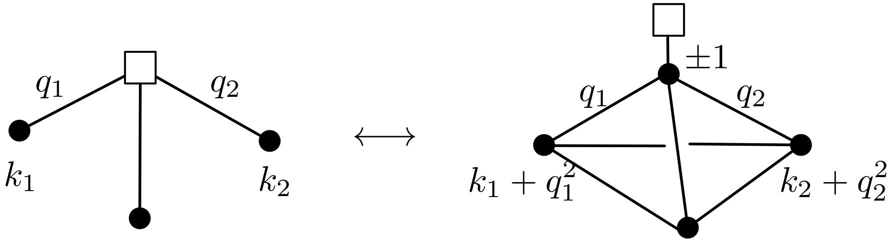


Application

- Examples

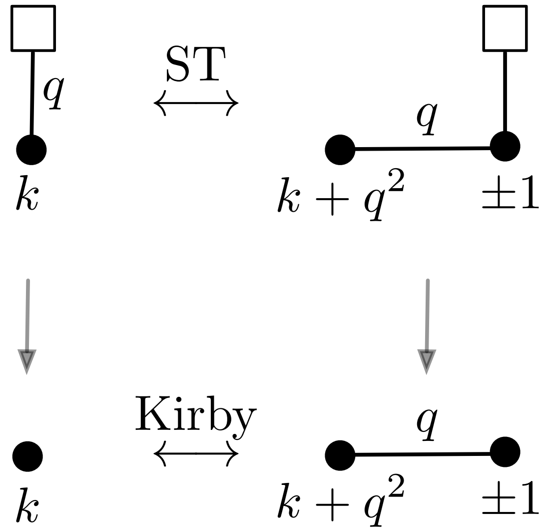






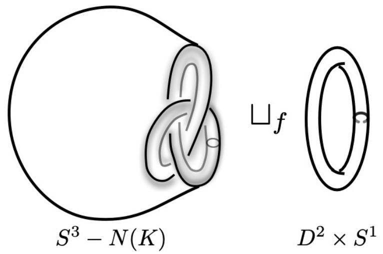
Decoupling

- After decoupling the matter, ST-moves reduce to Kirby moves.

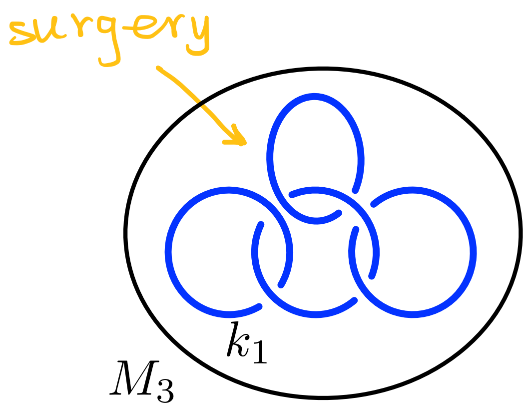


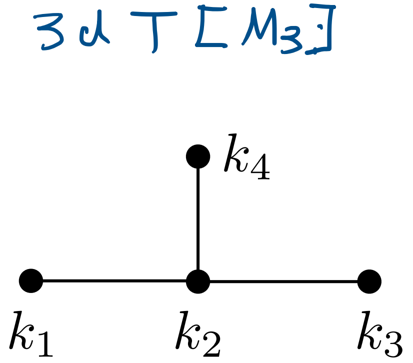
- Why is it a Kirby move?

Closed 3-manifolds, T[M_3] theories



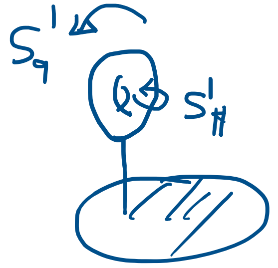
- In Gadde, Gukov, Putrov "Fivebranes and 4-mfds" [1306.4320]. Pure plumbing theories are realized by wrapping a single M5-brane on closed three-manifolds.



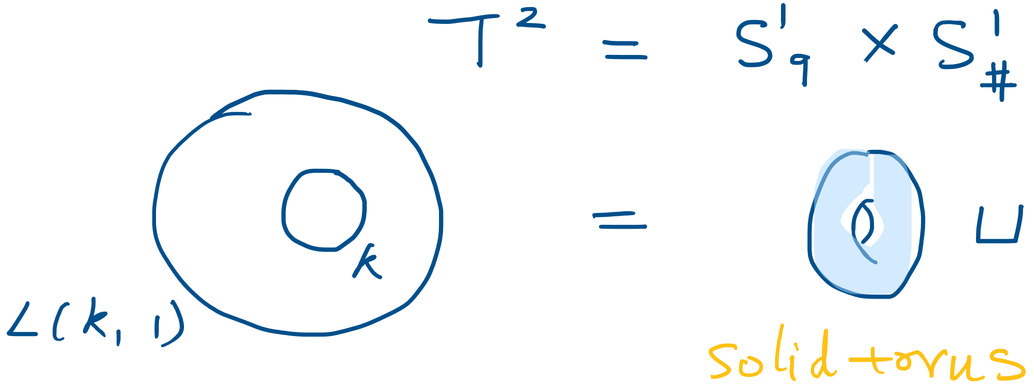


Linking number = CS levels

M-theory picture and brane wbes

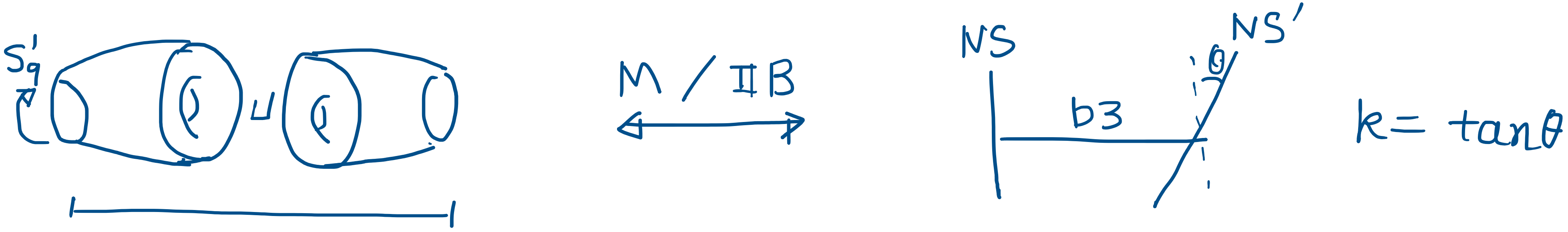


- Lens space L(k,1) in M-theory should be elliptically fibered:



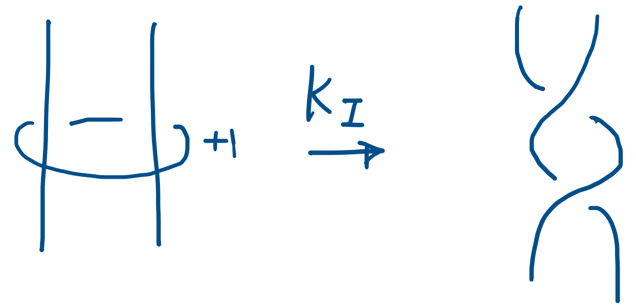
		$S^1 \times \mathbb{R}^2$			N_{345}			$I_6 \times T^2_{9\sharp}$		
11d	branes	0	1	2	3	4	5	6	9	♯
M-theory	N_c M5	0	1	2				6	9_A	♯
IIA	N_c D4	0	1	2				6	9_A	
IIB	N_c D3	0	1	2				6		
IIA	D0									♯
IIA	D6	0	1	2	3	4	5		9_A	
IIB	D5 $\xrightarrow{S}$ NS5	0	1	2	3	4	5			
M-theory	M5″	0	1	2	3	4			9_A	
IIA	NS5″	0	1	2	3	4			9_A	
IIB	NS5″ $\xrightarrow{S}$ D5	0	1	2	3	4			9_B	
M-theory	M2	0					5		9_A	
IIB	D1 $\xrightarrow{S}$ F1	0					5			

- Putting a M5-brane on it duals to a 3d brane web of U(1)_k

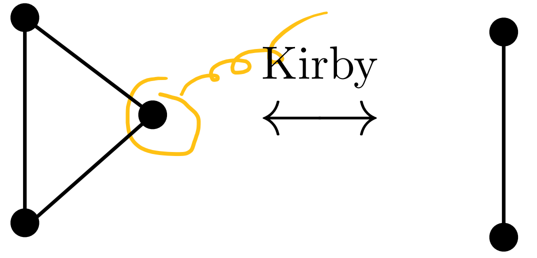


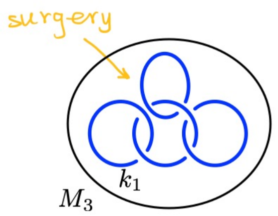
Kirby moves

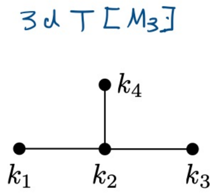
- For 3-mfds, the Kirby-I move is an equivalent surgery.



- Kirby moves are integrating in/out gauge node U(1)_k:

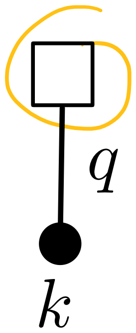






Question: how to add matters?

- Does the matter □ corresponde to some structures on the 3-mfds?

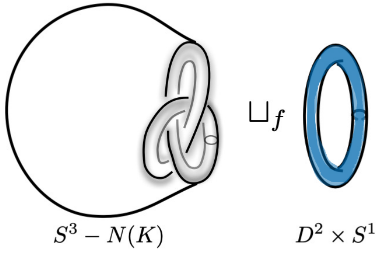


← what is this guy ?

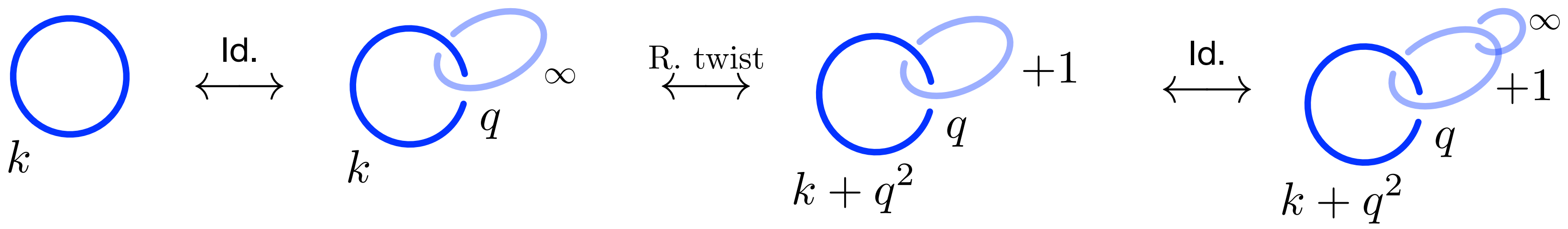
* Is it real ?

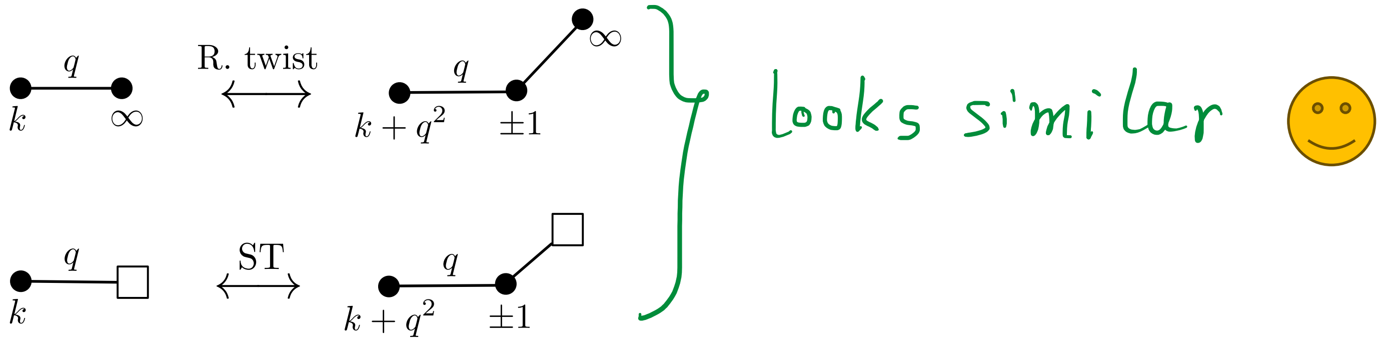
* How to geometrically realize it ?

Identical surgery and R. twist



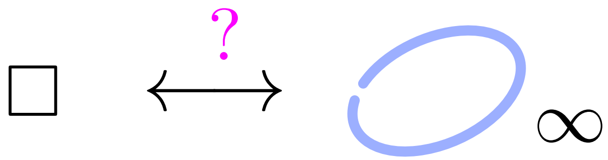
- The identical surgery, and rational equivalent surgery





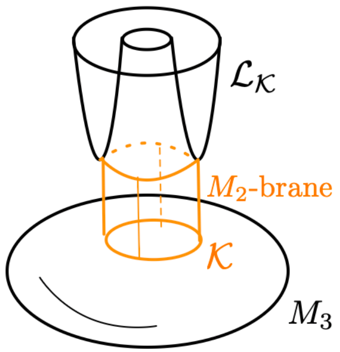
An observation

- Does the matter node correspond to the identical surgery circle?

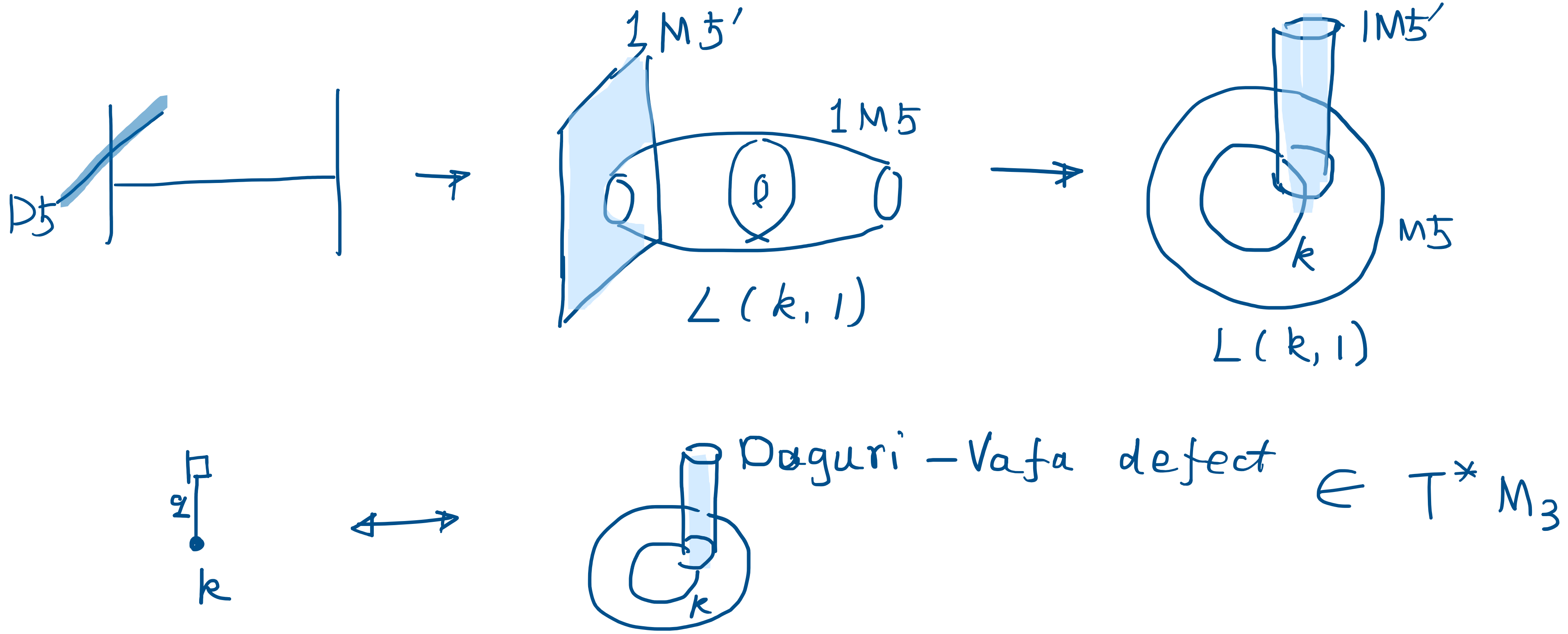


- However, the identical circle can be ignored on 3-mfds and is not physical, while the matter field is physical.
- So, we should do something to make it physical.

Ooguri-Vafa defect -> matter

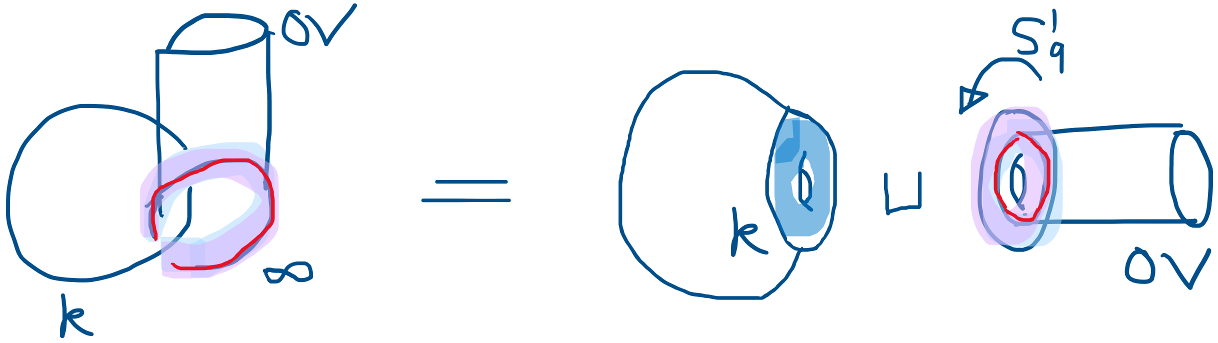


- Adding D5-branes leads to matters



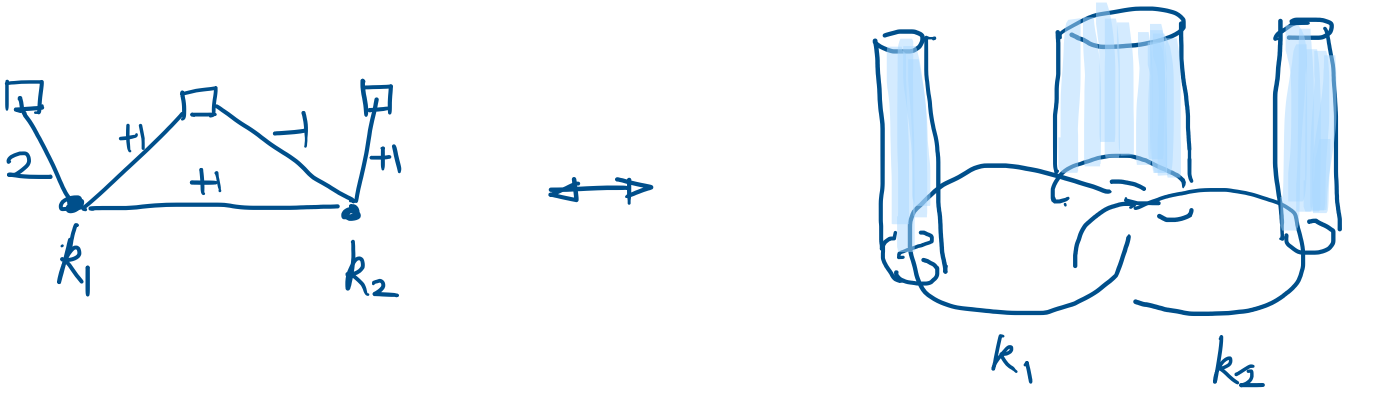
The 1 M5-brane on OV defect in the cotangle bundle realizes a matter field.

- The neighborhood of the intersection is always an identical surgery circle:



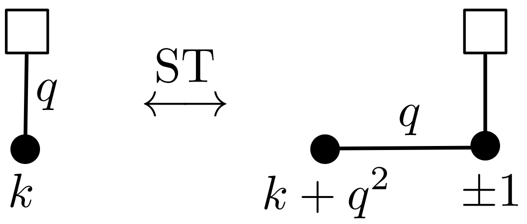
- The matter circle/intersection has to be S^1_q

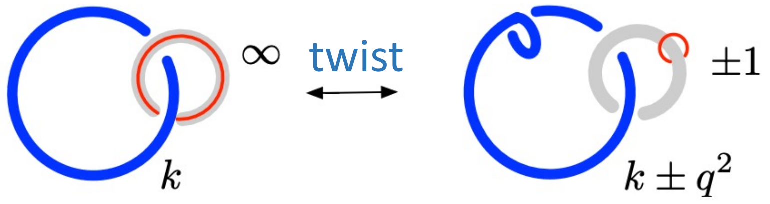
- Example:



ST-move and 3-mfds

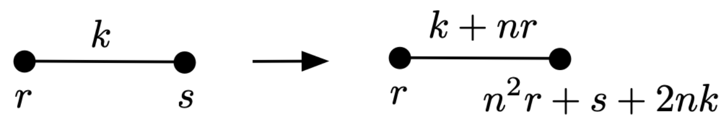
- ST-move is a particular Kirby-I move with an OV-defect/brane:

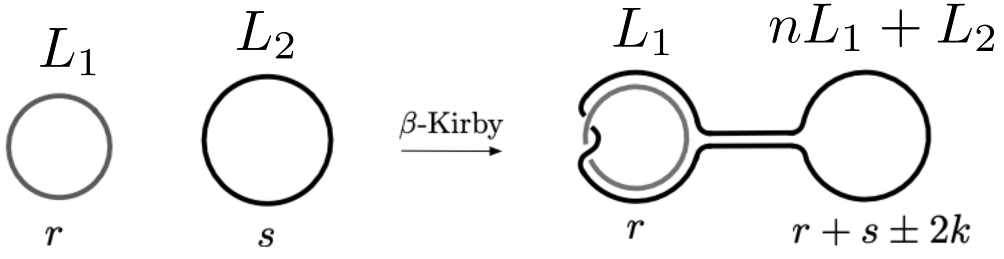




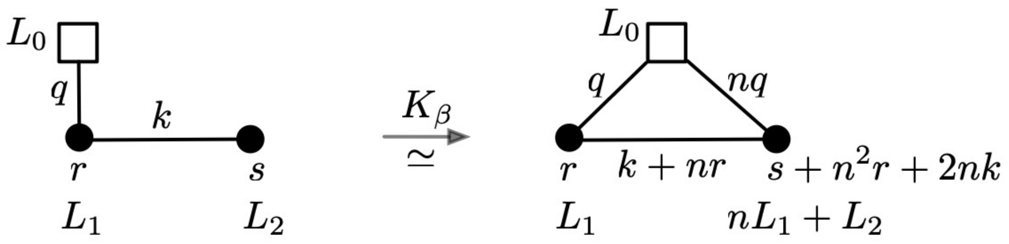
Kirby-II: handle-slides for gauge circles

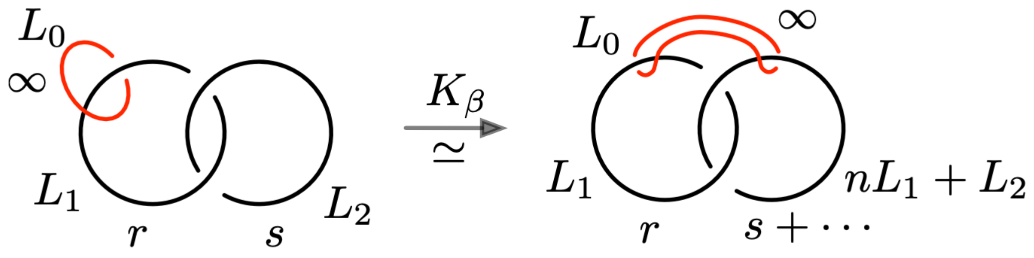
- Kirby-II is a connected sum of surgery circles:





- In the presence of the OV defect (or matter):

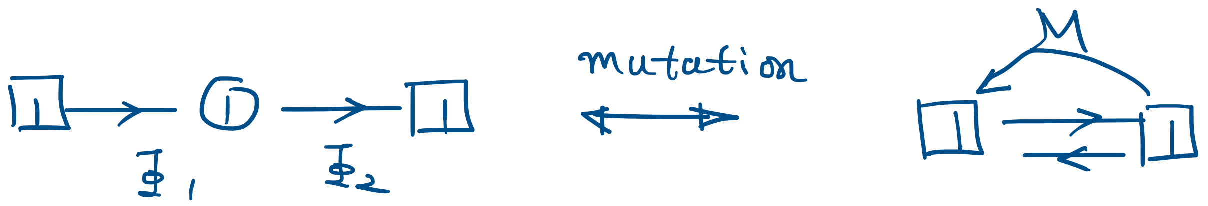




- Kirby-II is a linear sum of scalar fields: $\phi_1' = n\phi_2 + \phi_1 \,, \phi_2' = \phi_2$

Seiberg duality

- SQED-XYZ duality

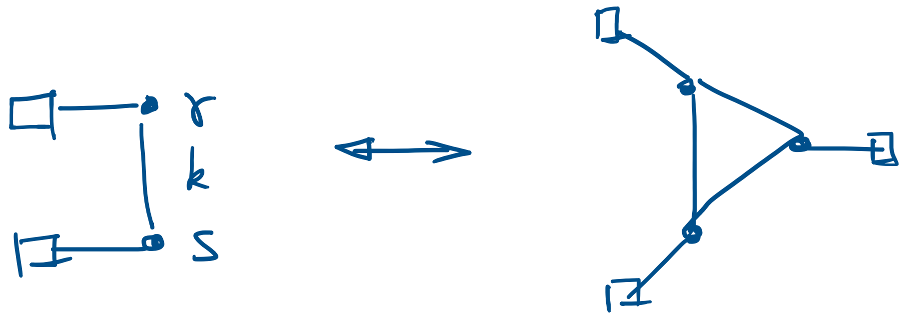


- Superpotential

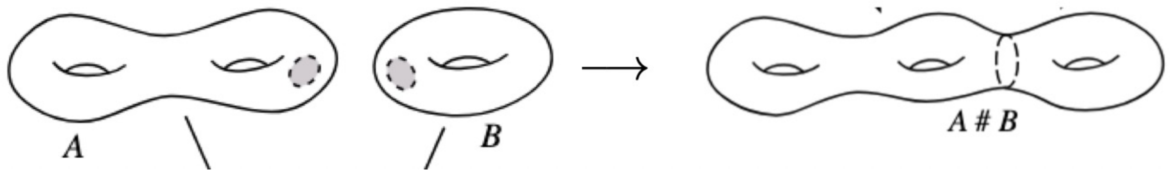
$W = 0$ $\qquad\qquad$ $W = \Phi_1 \Phi_2 M.$

- Flavor symmetry $U(1)_1 \times U(1)_2$

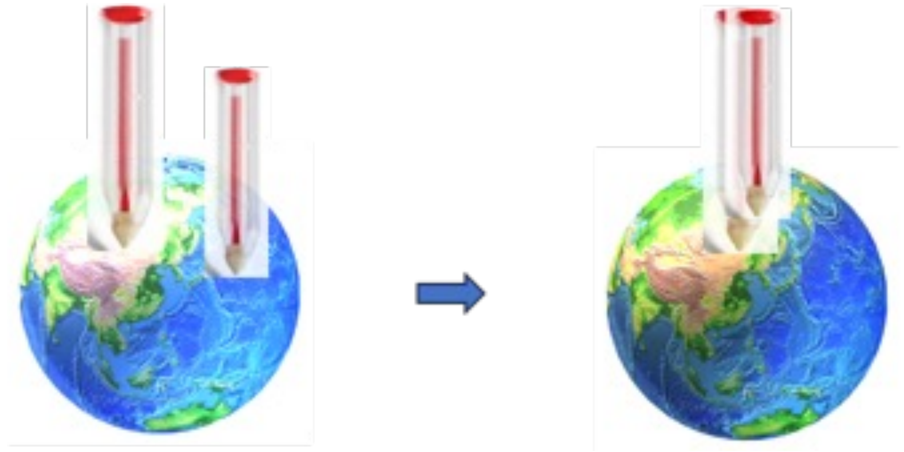
- Gauging flavor symmetries leads to superpotential triangles.

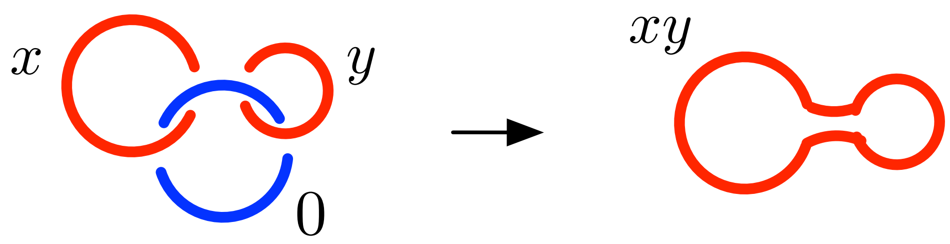


Connected sum



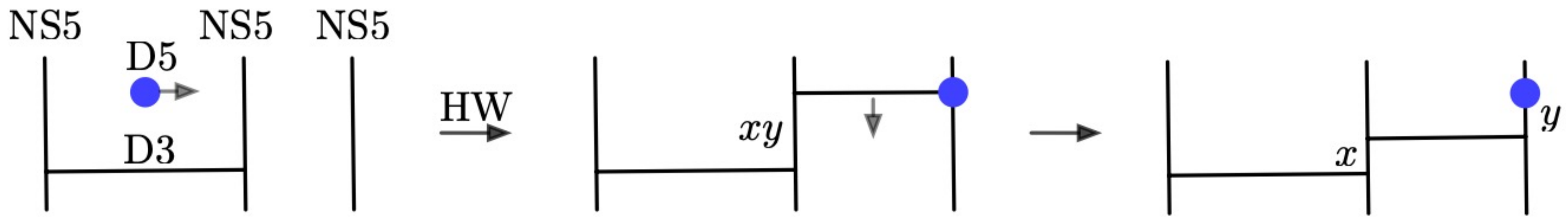
Handle slides for matter circles:





Superpotential arises from connected sums and gauging.

HW moves interpretation



Everything arises from a fusion identiy, which related to quantum group.

$$\frac{(xy,q)_n}{(q,q)_n} = \sum_{k=0}^{n} \frac{(x,q)_{n-k}}{(q,q)_{n-k}} \cdot \frac{(y,q)_k}{(q,q)_k} x^k$$

Open question

- Non-abelian theories.

Thank you !